Microwave Filter Analysis Using a New 3-D Finite-Element Modal Frequency Method

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***Abstract*— A new finite element modal frequency method is presented and shown to be advantageous for the analysis of microwave filters. The method analyzes a finite-element model of a filter by first computing the eigenmodes of the three-dimensional (3-D) structure. The computed eigenvalues are shown to reliably determine *all* of the resonant frequencies in a frequency range; the filter design can be changed until the desired resonant frequencies are computed. Finally, the eigenvectors are used as basis functions to compute the frequency response of the filter, thereby achieving a speedup that increases with the number of frequencies analyzed. Two filters analyzed in this paper show speedups ranging from 1.39 to 4.04, and their computed S-parameters agree closely with measurements.**

***Index Terms*—Eigenvalues/eigenfunctions, finite-element methods, microwave filters, resonator filters, scattering parameters.**

I. Introduction

MICROWAVE filters are often made of electromagnetic resonators that are tuned and coupled to obtain the desired frequency response. The filter design process is highly dependent on accurate knowledge of the resonant frequencies of the filter.

Finite elements are increasingly being used to analyze microwave components in three dimensions, including filters [1]–[4]. The S-parameters of a component are computed by solving a large finite-element matrix equation, where the right-hand side (RHS) is an excitation at each one of its ports. The matrix changes with each frequency, and thus the large matrix equation must be solved anew at each frequency analyzed. Hence, obtaining the frequency response of a filter with many sharp resonances can entail computations at dozens or hundreds of frequencies, which is very expensive.

To save computer time in such finite-element computations, several schemes for frequency interpolation have been used [5]. Recently, the asymptotic waveform evaluation (AWE) method has gained favor. It is typically based on Taylor's series, recursion formulas, and Padé approximants [6], [7]. AWE has limitations, however, especially for filters with many resonances, and thus other methods such as Padé via Lanczos [8] are now emerging.

This paper describes a new finite-element method (FEM) that aids the design of microwave filters. It is called the electromagnetic finite-element modal frequency method because it is based upon first computing the resonant electromagnetic modes. If the resonant frequencies do not correspond to those desired for the filter, then the designer should not spend any time computing the frequency response. Instead, the geometry and/or materials can be changed to tune the resonant frequencies to the desired values. The final computational step of the new method then quickly obtains the filter S-parameters at a large number of frequencies.

This paper begins with the theory of the new method, starting with a review of real eigenvalue extraction. The authors show how *all* of the modal eigenvectors in a frequency range can be found and then used as orthogonal basis functions in place of the usual finite-element shape functions. The authors then examine the computation of S-parameters of microwave components, and find a few special requirements when the modal frequency method is used. Finally, the authors show results for typical microwave filters modeled with H1-curl edge finite elements.

II. Electromagnetic Finite Element Modal Frequency Method

A. Comparison of Direct and Modal Methods

The computing cost of dynamic structural finite-element analysis is often reduced by first computing structural resonant modes. Structural engineers often perform dynamic analysis, including frequency response, transient response, and complex eigenvalue analysis, by *modal methods* [4], [9] rather than by *direct methods*.

In direct methods, whether structural or electromagnetic, the piecewise (low-order polynomial) finite-element shape functions are used to describe the solution in the form of a large matrix equation with potentials or fields as the direct unknowns $\{u\}$. In modal analysis, basis functions representing resonant behavior are used instead. In modal dynamics the final solution $\{u\}$ is expressed as a linear combination of the orthogonal eigenvectors $\{\phi_i\}$ found in real eigenvalue analysis

$$\{u\} = \sum_{i=1}^{m} \{\phi_i\} q_i. \tag{1}$$

This approach can have a tremendous advantage. Instead of describing a three-dimensional (3-D) solution in terms of typically 10 000 or so direct degrees of freedom in $\{u\}$, the solution may now be often described by only a few dozen mode amplitude degrees of freedom q_i. The disadvantage, of course, is that the 3-D resonant modes must first be computed. Overall, the modal method can have a significant advantage in frequency response analyses in which a large number of

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frequencies are requested, or in transient analyses involving large numbers of time steps and/or time step size changes.

The modal transformation given in (1) can also be written in matrix form as

$$\{u\} = [\phi]\{q\} \tag{2}$$

where the matrix $[\phi]$ is made up of m columns of individual orthogonal eigenvectors $\{\phi_i\}$, and the vector $\{q\}$ contains all of the coefficients of (1). If there are n direct degrees of freedom in a problem (the length of the column vector $\{u\}$), then $[\phi]$ is an $(n \times m)$ matrix. This transformation can be highly accurate when all n eigenvectors of the system are used. In many cases only a small approximation is introduced if a limited number of eigenvectors in a specified frequency range is used. The frequency range should include all modes that are expected to respond during dynamic analysis.

B. Finding All Resonant Modes in a Frequency Range

The eigenvectors $\{\phi_i\}$ of (1) are found using Sturm sequence techniques as follows. The eigenvalue problem to be solved is

$$([K] - \lambda_i[M])\{\phi_i\} = 0 \tag{3}$$

where the matrices $[K]$ and $[M]$ for electromagnetics are reluctance and permittance matrices, respectively [4], [10]. The eigenvalues to be found are real numbers λ_i equaling the square of each resonant angular frequency. Note that each eigenvalue has a corresponding eigenvector $\{\phi_i\}$.

The solution of (3) is obtained here by the widely used Lanczos algorithm [4]. It begins with an inverse power iteration in which orthogonality conditions are imposed among the vectors produced by iteration. The normalization coefficients from the iteration are then used to construct a tridiagonal matrix with the same eigenvalues as the original. Once eigenvalues are extracted from the tridiagonal matrix, eigenvectors are easily constructed through a second iteration and orthogonalization process.

The required decomposition of the initial symmetric positive definite matrix $[K - \lambda_0 M]$, where λ_0 is a shift value, provides an excellent opportunity for monitoring the iteration process to assure that all roots are found within a specified frequency interval. Consider the initial decomposition

$$[K - \lambda_0 M] = [L]^T[D][L]. \tag{4}$$

Let $p_j(\lambda)$ denote the characteristic polynomial of the leading principal $(j \times j)$ submatrix of $[K - \lambda_0 M]$. The roots of such a polynomial represent the eigenvalues of the submatrix. The sequence of polynomials $\{p_0, p_1, \cdots, p_n\}$ is called a Sturm sequence. By Cauchy's interlace theorem, it is known that the roots of the polynomials in such a sequence are *interlaced*. Consider the sequence of numbers generated by evaluating the characteristic polynomials at some shift value λ_0 for increasingly larger submatrices

$$\{p_j(\lambda_0),\, j = 0, 1, \cdots, n\}. \tag{5}$$

The number of sign agreements in this sequence is equal to the number of roots (i.e., eigenvalues of the original matrix) below λ_0. Estimates of this kind are extremely useful in finding roots within selected intervals.

In general, the evaluation of characteristic polynomials is a difficult task. Fortunately, the decomposition in (4), which must be performed in any event to implement the basic iteration algorithm, can also provide the necessary information. It can be shown that the characteristic polynomial of the original matrix is related to the determinant of the j-size submatrix of the factor diagonal matrix $[D]$

$$(-1)^j p_j(\lambda_0) = d_1 d_2 \cdots d_j = \det[D_j]. \tag{6}$$

From this, it follows that the number of roots below λ_0, i.e., the Sturm number $S(\lambda_0)$, is simply equal to the number of negative terms on the factor diagonal. Given the Sturm numbers at two points, λ_a and λ_b, the number of roots in the interval $[\lambda_a, \lambda_b]$ is just the difference between Sturm numbers. Such information, which is readily available as a result of the iteration process, allows the algorithm to easily track the number of roots within selected intervals.

It sometimes happens that the basic iteration algorithm fails, for some reason, to find all of the roots within a selected interval. When this occurs, a binary search is conducted based on Sturm numbers to isolate the missing roots. Assume for the sake of discussion that only one root has been missed. The interval is bisected and the Sturm number at the midpoint is computed. On the basis of this information, and the location of the roots already found, it is possible to decide which half of the interval contains the missing root. This procedure is continued until the missing root is isolated within a sufficiently small subinterval. It may then be extracted by iteration.

C. Using the Modes to Obtain Frequency Response

Electromagnetic frequency response can be computed as follows. The equation for direct frequency analysis is given by the usual finite-element matrix equation [4], [10]

$$\{-\omega^2[M] + j\omega[C] + [K]\},\, \{u\} = \{P\}. \tag{7}$$

For electromagnetics, $[M]$ is the permittance matrix (proportional to permittivity), $[C]$ is the conductance matrix (proportional to conductivity), and $[K]$ is the reluctance matrix (inversely proportional to permeability). For the edge finite elements used here, $\{u\}$ consists of edge magnetic vector potentials $\overrightarrow{A}$. From $\{u\}$, the magnetic fields are easily found by a curl operation and the electric fields by multiplication by $-j\omega$. $\{P\}$ is the excitation vector. For microwave S-parameter computations, $\{P\}$ is located at ports. Assuming a 3-D finite-element model of a microwave component with thousands of edge unknowns, the unknown $\{u\}$ vector has thousands of degrees of freedom. Note that the left-hand matrix changes with frequency and thus solution time is proportional to the number of frequencies analyzed.

Instead of solving (7) directly, one can substitute (2) into (3), obtaining

$$-\omega^2[M][\phi]\{q\} + j\omega[C][\phi]\{q\} + [K][\phi]\{q\} = \{P\}. \tag{8}$$

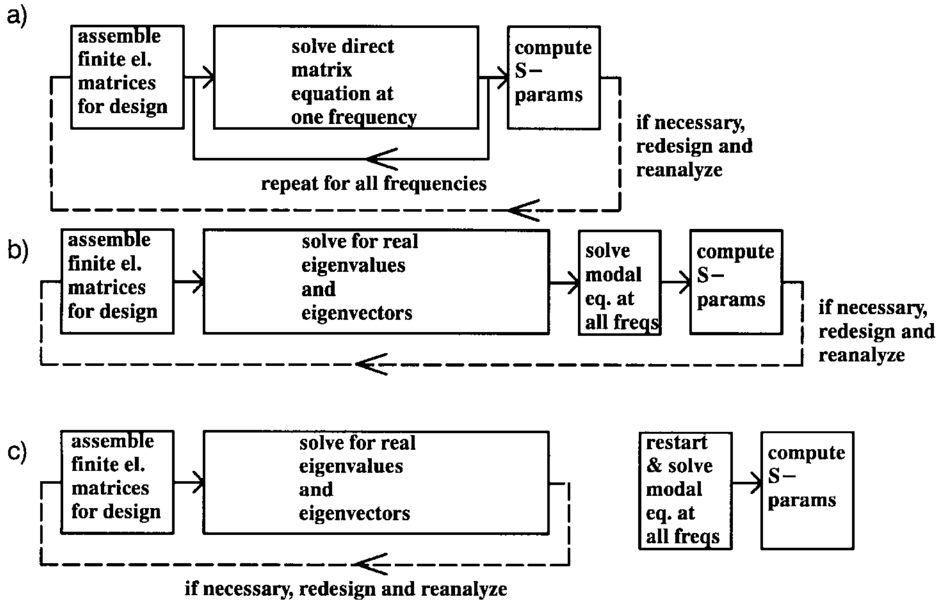


Fig. 1. Basic schematics and relative computer times for alternative design/analysis FEM's. Block widths are roughly proportional to typical computer times. (a) Direct frequency. (b) Modal frequency. (c) Modal frequency with restart.

Premultiplying both sides of (8) by $[\phi]^T$ obtains

$$-\omega^2[\phi]^T[M][\phi]\{q\} + j\omega[\phi]^T[C][\phi]\{q\} + [\phi]^T[K][\phi]\{q\} = [\phi]^T\{P\} \tag{9}$$

which can be rewritten as the *modal frequency equation*

$$(-\omega^2[m] + j\omega[c] + [k])\{q\} = \{p\} \tag{10}$$

where the three new *modal* matrices are

$$\begin{aligned}[m] &= [\phi]^T[M][\phi]\\ [c] &= [\phi]^T[C][\phi]\\ [k] &= [\phi]^T[K][\phi]\end{aligned} \tag{11}$$

and the modal right-hand excitation vector is

$$\{p\} = [\phi]^T\{P\}. \tag{12}$$

The solution of (10) may become trivial under certain cases. If the $[M]$ and $[K]$ matrices in (7) are any real symmetric matrices and $[K - \lambda_0 M]$ is positive definite, then the eigenvector matrices $[\phi]$ have properties such that (10) becomes [11]

$$-\omega^2[m_d]\{q\} + j\omega[c]\{q\} + [k_d]\{q\} = \{p\} \tag{13}$$

where the subscript d indicates a diagonal matrix. Now if $[c]$ can be shown to be diagonal, then (13) reduces to a trivial *uncoupled* matrix equation, where each unknown q can be solved independently by the scalar equation

$$(-\omega^2 m_i + j\omega c_i + k_i)q_i = p_i. \tag{14}$$

Obviously, if the conductance matrix $[C]$ is zero, then the uncoupled solution (14) may be used.

The coupled solution (10) for all *lossy* electromagnetic devices will be used, because they either have nonzero $[C]$ or complex $[M]$ or $[K]$ matrices. However, because the size of the vector $\{q\}$ is usually much smaller than the size of the original unknown vector $\{u\}$, the solution of (10) is usually inexpensive. Thus, it can be efficiently accomplished at a large number of frequencies.

When the effects of conductivity are very large, the modes computed using real eigenvalue analysis may not adequately span the solution space. In such cases, the finite-element software must use a large number of eigenvectors or must resort to direct solution methods. However, most microwave filters have low losses and thus the modal frequency method is applicable. It is especially attractive for filters because their resonant frequencies should usually be checked first by real eigenvalue analysis, and thus the eigenvectors $\{\phi_i\}$ of (1) and (3) are already computed and saved on a database.

The proposed modal frequency method for microwave devices is compared with the direct frequency method in Fig. 1. Fig. 1(a) shows that if many frequencies must be analyzed in the direct method, then its computer time is essentially proportional to the number of frequencies. Fig. 1(b) shows that eigenvalue analysis is the most expensive part of the modal frequency method, and thus its total computer time is almost unaffected as the number of frequencies analyzed is increased. Fig. 1(c) shows that if eigenvalues (resonant frequencies) are of interest as in the case of most microwave filters, then often the best design method is to make any design changes after eigenvalue analysis, redesign until the desired eigenvalues are obtained, and finally restart to obtain the S-parameters versus frequency. This design procedure could be used inside a circuit optimization algorithm, and the speed gained by the modal frequency method would speed up the optimization.

III. S-Parameter Computation Method

The major requirement for using the new modal frequency response method for microwave filters is that it obtain accurate

S-parameters and the associated 3-D electromagnetic fields. To do so, some modifications must be made to the usual S-parameter computation procedures.

S-parameters can be computed as follows whether or not ports are matched [2]. The fields near a port of a microwave circuit can be expressed by

$$\vec{E}(x, y, z, t) = E_\tau(x, y)e^{-\gamma z}e^{+j\omega t}\hat{\tau} + E_z(x, y)e^{-\gamma z}e^{+j\omega t}\hat{z} \quad (15)$$

$$\vec{H}(x, y, z, t) = H_\tau(x, y)e^{-\gamma z}e^{+j\omega t}\hat{\tau} + H_z(x, y)e^{-\gamma z}e^{+j\omega t}\hat{z} \quad (16)$$

where τ indicates the transverse components (in the plane of a port) and z indicates the longitudinal (normal) component.

In order to compute S-parameters, the transverse components must be known. They can be expressed in terms of forward-traveling waves a and backward traveling waves b without loss of generality (where the sign of the b waves is different for $\overline{\vec{E}}$ and $\overline{\vec{H}}$ fields so that power flows in the proper direction)

$$(\vec{E})_\tau = (ae^{-\gamma z} + be^{+\gamma z})\vec{e}(x, y) \quad (17)$$

$$(\vec{H})_\tau = (ae^{-\gamma z} - be^{+\gamma z})\vec{h}(x, y). \quad (18)$$

Note that the transverse components can be represented by a complex amplitude and the real transverse eigenvector modal fields $\vec{e}$ and $\vec{h}$ computed using an analysis of the modal fields at a port (e.g., a waveguide TE_{10} mode, etc.). In general, both transverse and longitudinal components can exist, but only the transverse components are needed for S-parameter computation. The transverse modal port vector field of a lossless port can be represented by a pure real vector field without loss of generality or phase information. Hence (17) and (18) can be rewritten as

$$E_\tau\vec{e}(x, y) = (ae^{-\gamma z} + be^{+\gamma z})\vec{e}(x, y) \quad (19)$$

$$H_\tau\vec{h}(x, y) = (ae^{-\gamma z} - be^{+\gamma z})\vec{h}(x, y) \quad (20)$$

where E_τ and H_τ are unitless factors. Finally, adding and subtracting (19) and (20) gives

$$a = \frac{E_\tau + H_\tau}{2}$$
$$b = \frac{E_\tau - H_\tau}{2}. \quad (21)$$

Assuming a two port microwave circuit for clarity, two evaluations are required. One has its excitation at port one and the other has its excitation at port two. For an N-port, this pattern continues until all N ports have been excited individually for each mode of interest, thereby generating multiple matrix equations. The S-parameter matrix equations for the two port are

$$\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} \quad (22)$$

$$\begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}. \quad (23)$$

Combining them gives

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}. \quad (24)$$

Hence, one obtains the expression for the S-parameters

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1}. \quad (25)$$

A similar relation applies to a multimode N-port circuit.

The above S-parameter equations require that accurate E_τ and H_τ are used in (21). Thus, accurate tangent fields at the ports are needed.

Obtaining accurate tangent fields at the ports is difficult in the modal frequency method, because the 3-D real eigenvalue analysis assumes either perfect electric conductor (PEC) or perfect magnetic conductor (PMC) boundary conditions at the ports (and on all other outer boundaries). PEC conditions cause E_τ to be zero, while PMC conditions cause H_τ to be zero. Neither condition is correct at excited ports.

While other techniques to resolve the port boundary condition problem are under investigation, in this paper one has assumed PMC boundary conditions at the port terminations during the 3-D eigenvalue analysis, but has shifted the port terminations outwards from their original locations by mesh extension. The S-parameter computations use E_τ and H_τ evaluated at the original (unshifted) port locations. H_τ is computed with much greater accuracy because of the shift in the PMC port boundary condition, and hence (21) and (25) obtain more accurate S-parameters. In the software used here [12], the evaluation of E_τ and H_τ is automatically carried out three to five finite-element layers away from the port PMC boundary whenever the modal frequency method is used. Note that the same mesh with extended terminations is used for both steps of the modal frequency solution.

The real 3-D eigenvalue analysis must also have proper boundary conditions on all electrically conductive walls. If they are PEC's, then the tangent component of $\overline{E}$ is set to zero and the proper eigenvalues and eigenvectors are found. However, 2-D conducting finite elements that model resistive wall losses in direct frequency analysis [10] require that tangent E be unconstrained, and thus cannot be used in modal frequency analysis. Instead, one could estimate such wall losses using perturbation theory.

IV. EXAMPLES

The modal frequency solution method is applied in this section to the analysis of two low loss microwave filters. The method has been implemented [12] such that computed eigenvectors are stored on a database and are available via restart techniques for modal frequency solutions at any desired frequencies.

The two filters analyzed here have also been analyzed by the direct frequency FEM [13]. They are types of dielectric resonator filters that use evanescent waves to couple to two external ports. While design iterations will not be discussed for these example filters, the software also includes fully parameterized solid geometry and meshing to aid the evaluation of multiple design alternatives.

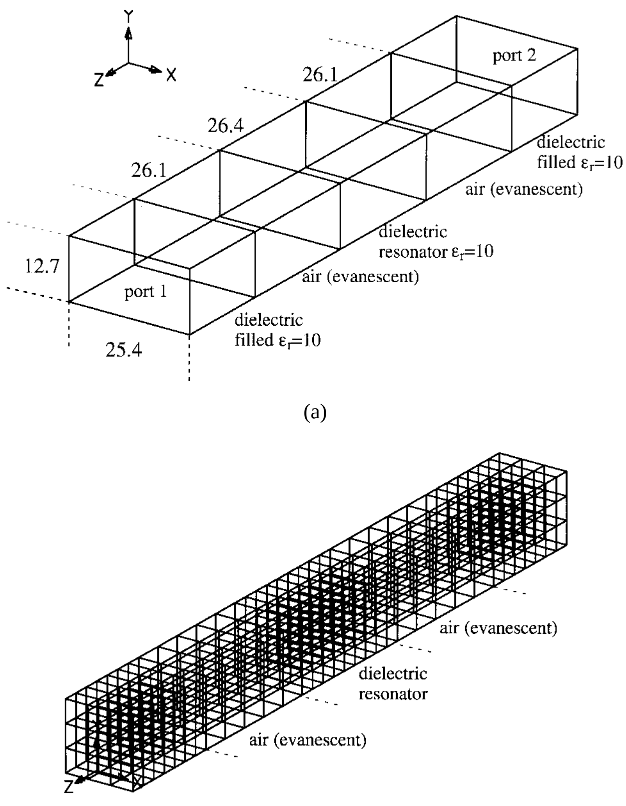


Fig. 2. Filter with rectangular dielectric block that fills metal walled waveguide. (a) Dimensions in mm (26.4 has been tuned by +3.6% from measured length). (b) finite-element model of one-half.

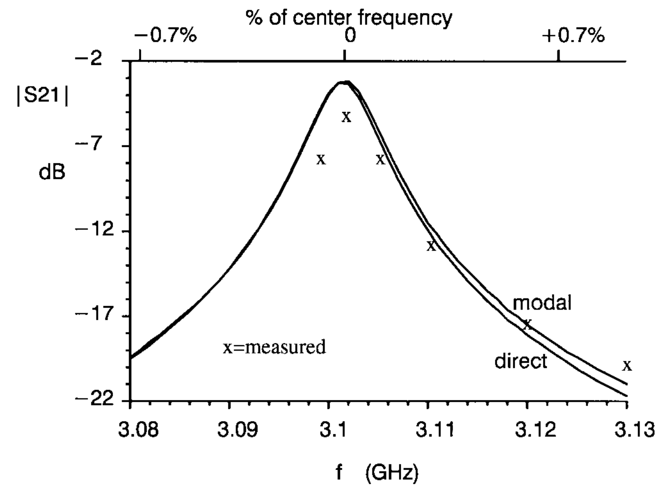


Fig. 3. $|S_{21}|$ for rectangular filter of Fig. 2. Curves are computed by direct and modal methods, while data points are measurements.

TABLE I
COMPUTER TIMES (SECONDS) TO COMPUTE S-PARAMETERS FOR GIVEN DESIGN OF RECTANGULAR FILTER OF FIG. 1

Number of	DIRECT METHOD		MODAL METHOD		Speedup
frequencies	Solving matrix eq.	Total	Solving matrix eq.	Total	(Total)
51	361	531	1.3	381	1.39
101	718	928	2.5	439	2.11

A. Cutoff-Coupled Rectangular Dielectric Resonator Waveguide Filter

Fig. 2(a) shows a bandpass filter made of a length of rectangular metal walled waveguide. It is filled with a dielectric material at its two ports and at a central resonator section. Between the dielectric filled sections are two air filled sections. The air filled sections contain evanescent waves, while the dielectric sections have a relative permittivity of ten and contain propagating TE_{10} waves. The lengths of the two adjacent air sections determine the coupling and loaded quality factor Q_L of the filter, much as the size of coupling holes in the metal walls of an ordinary resonant cavity filter determine its Q_L.

The resonant frequency of the filter is related to the length d of the dielectric resonator, which here is 26.1 mm. The theoretical equation satisfied at resonance is [14]

$$\beta d + \arctan\left(\frac{2\alpha\beta}{\alpha^2 - \beta^2}\right) = p\pi \tag{26}$$

where α is the attenuation constant in the evanescent sections and β is the propagation constant in the propagating sections. The RHS has the integer p, which is here assumed to be one. For the waveguide shown in Fig. 2(a), (26) is obeyed at approximately 3.1 GHz (when $\alpha = 105$/m and $\beta = 164$/m).

Fig. 2(b) shows the finite-element model developed for the filter of Fig. 2(a). Due to symmetry of the 25.4 $\times$ 12.7 mm waveguide, only a half model (12.7 $\times$ 12.7 mm) was analyzed. The model includes the dielectric loss tangent of 7 $\times 10^{-4}$ in the dielectric filled sections, which causes both matrices $[M]$ in (7) and $[m]$ in (10) to become complex. The 3-D finite elements are all H1-curl edge hexahedrons. Including the shifted ports for the modal solution, the model has 3738 direct (edge) degrees of freedom.

Fig. 3 shows the transmission coefficient $|S_{21}|$ near the resonant frequency of 3.10 GHz. Measured data points are indicated at seven frequencies. Also shown are two curves computed at 51 frequencies over the range from 3.08 to 3.13 GHz. The computed curves include the dielectric loss but not wall loss, which can be shown to be small compared to the dielectric loss [13], [14]. Unfortunately, the experimental filter was discarded many years ago, and no measurements of its reflection coefficient S_{11} are available.

The two curves of Fig. 3 were obtained by direct frequency and modal frequency methods for comparison. Note that the agreement is excellent, especially near resonance. Moreover, Table I shows that the new modal frequency method offers a substantial time advantage over the customary direct method using sparse decomposition. The computer time speedup of 1.39 for 51 frequencies is increased to 2.11 when 101 frequencies are analyzed; the speedup will continue to increase with the number of frequencies analyzed. All times in Table I are for a Hewlett-Packard 735/125 workstation.

The modal results of Table I were obtained using 50 modes. Fewer modes were found to yield greater speedups at some sacrifice of accuracy at frequencies above 3.13 GHz. The

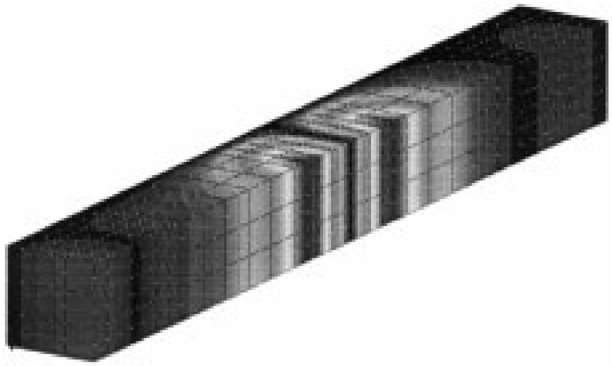
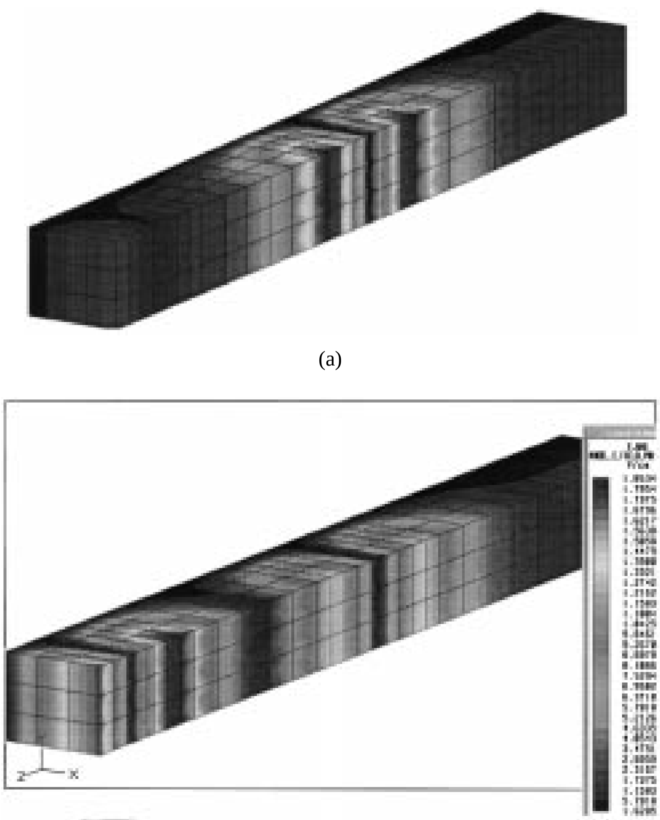


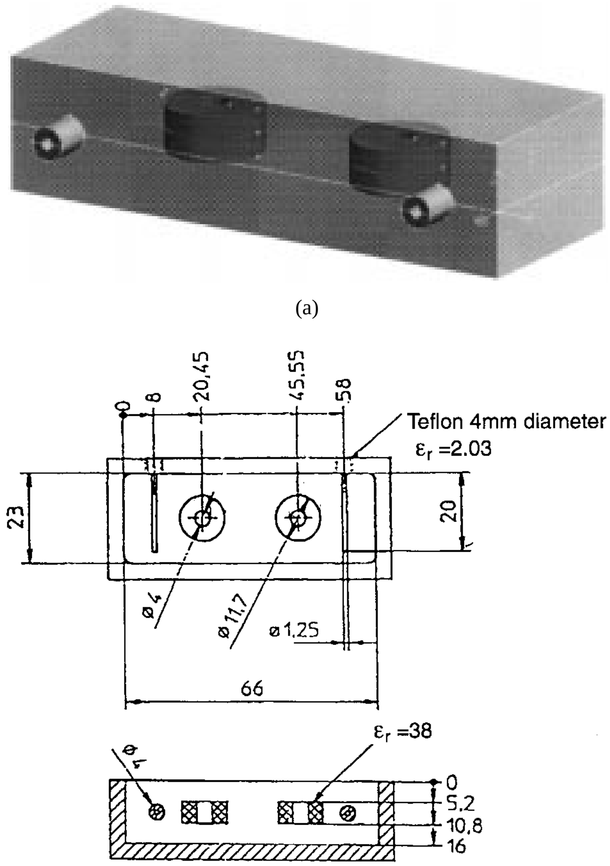
Fig. 4. Electric field computed by modal frequency method for rectangular filter of Fig. 2 at 3.10 GHz.



(a)

(b)

Fig. 5. Electric fields computed by direct frequency method for rectangular filter of Fig. 2. (a) at resonant frequency of 3.10 GHz and (b) 3.13 GHz (maximum field is now reduced by a factor of 50).



(a)

(b)

Fig. 6. Filter with two dielectric cylinders in a cutoff waveguide fed by two coaxial cables serving as ports 1 and 2. (a) Translucent 3-D view of solid geometry and (b) dimensions in mm and materials.

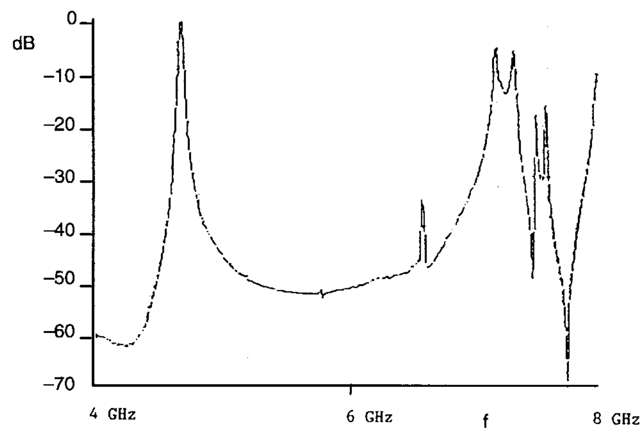


Fig. 7. Measured $|S_{21}|$ of cylindrical filter of Fig. 6.

modes used are the 50 modes with the lowest frequencies above 0.1 GHz. Modes near zero frequency should be excluded due to zero frequency spurious modes of edge elements and breakdown of the Lanczos decomposition. The modes used for Table I had frequencies ranging from 1.97 to 7.16 GHz; the sixth mode is at 3.10 GHz.

Fig. 4 shows the electric field computed by the modal method at 3.10 GHz. This shows strong resonance in the rectangular dielectric block. The maximum electric field is 19 739 V/m, and total electric energy (the volume integral of $\vec{E} \cdot \vec{D}/2$) is 30.972 J.

Fig. 5(a) shows the electric field computed by the direct method at 3.10 GHz, which is very similar to that of Fig. 4. The maximum electric field is altered slightly to 19 732 V/m, and total electric energy is now 30.974 J. Therefore the maximum E has been changed by −0.035%, and the average E has been changed by +0.0032%.

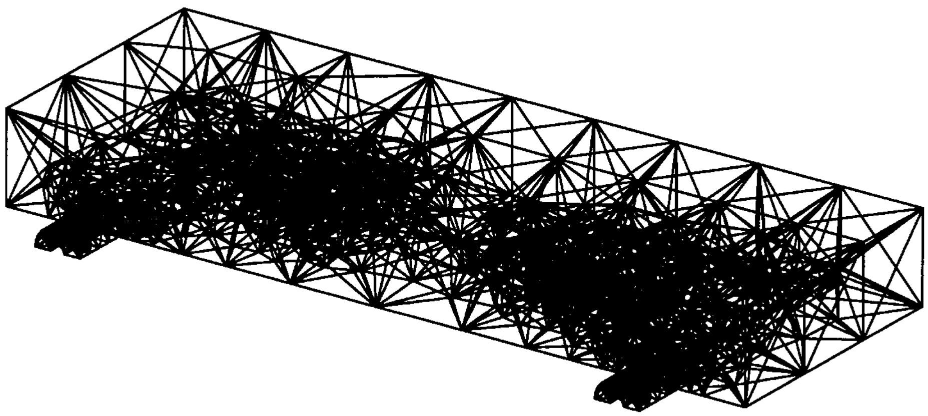


Fig. 8. Finite-element model of one-half of cylindrical filter of Fig. 6, consisting of 3503 tetrahedrons.

Fig. 5(b) shows the electric field computed at 3.13 GHz (away from resonance). Note that the field is rapidly attenuated from its input at the left port and is now much smaller in the resonator section.

B. Coax Fed Cylindrical Dielectric Resonator Filter

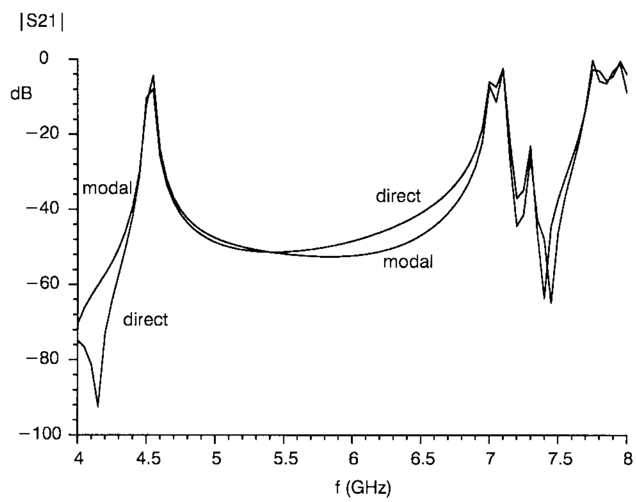
Fig. 6(a) shows a filter design with two cylindrical dielectric resonators. It is similar to filters designed for remote sensing satellites [13]. It has two coaxial cables acting as ports, and is designed to perform as a bandstop filter from 5 to 6.8 GHz. Fig. 6(b) shows the dimensions and materials of the filter. The two dielectric cylinders have a relative permittivity of 38. The rectangular waveguide connecting the two ports is operated in its evanescent TE_{10} mode.

Fig. 7 shows the measured transmission coefficient $|S_{21}|$ over the frequency range from 4 to 8 GHz. Note the resonance at 4.5 GHz and additional resonances at frequencies between 7 and 8 GHz. The peak near 6.6 GHz, however, appears to be too short to be a true resonance. Measurements of return loss S_{11} are unavailable. However, since this two port filter is assumed lossless and reciprocal, $|S_{11}|$ can be obtained from the relation that the sum of the squares of $|S_{11}|$ and $|S_{21}|$ must equal one.

Fig. 8 shows the finite-element model developed for the filter, which again is a half model due to symmetry. The 3-D finite elements are all H1-curl edge tetrahedrons. The model has a total of 24 190 direct degrees of freedom.

Fig. 9 shows two computed $|S_{21}|$ curves, obtained by direct and modal methods for comparison. Note that the two curves agree closely with each other, especially near the two resonant peaks. Far away from the resonances some discrepancies exist, but these are for magnitudes less than −40 dB. The eigenvector basis functions used in the modal method evidently cause it to lose some accuracy at frequencies far away from resonant frequencies. The modal results were obtained using all modes computed from 4.0 to 8.0 GHz. Over this range 12 modes, ranging from 4.06 to 7.96 GHz, were obtained. Fig. 10 shows the modal fields computed at 4.51 and 7.02 GHz.

The curves in Fig. 9 agree well with the measurements of Fig. 7 except in three ways. First, neither computed curve contains a resonant peak near the short peak measured near 6.6 GHz, which may, therefore, be a measurement artifact. Second, magnitudes below about −60 dB show disagreement; the cause has not been researched but may be due to noise floors on both calculations and measurements. Third, at the highest frequencies (near 8 GHz) there is evidently some frequency shift error in the computations due to the decrease in the number of finite elements per wavelength.



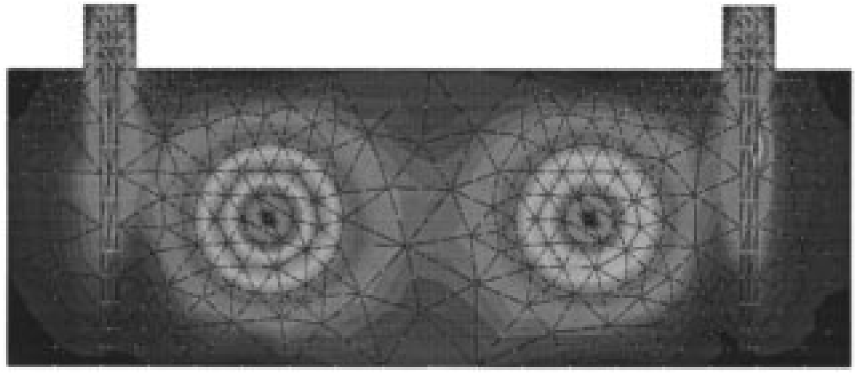
(b)

Fig. 9. $|S_{21}|$ computed for cylindrical filter of Fig. 6 at 81 frequencies.

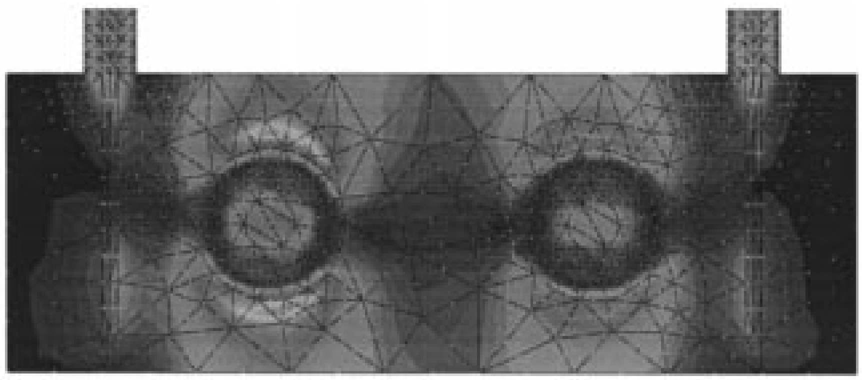
Table II shows once again that the new modal method offers a substantial time advantage over the customary direct method. The speedup of 2.17 for 41 frequencies is increased to 4.04 when 81 frequencies are analyzed. Because this filter is lossless, the modal matrix equation setup and solution times are even faster than for the lossy filter of Table I, and thus, results at additional frequencies are obtained with no additional computer time.

V. Conclusion

A new method has been derived that speeds up the analysis and design of microwave filters. Called the finite-element modal frequency method, it first uses the Sturm sequenced Lanczos method to reliably compute *all* of the real 3-D modes



(a)



(b)

Fig. 10. Computed modal fields for cylindrical filter. (a) Mode 3 at 4.51 GHz. (b) Mode 7 at 7.02 GHz.

TABLE II
COMPUTER TIMES (SECONDS) TO COMPUTE S-PARAMETERS FOR GIVEN DESIGN OF CYLINDRICAL FILTER OF FIG. 6

Number of frequencies	DIRECT METHOD Solving matrix eq.	Total	MODAL METHOD Solving matrix eq.	Total	Speedup (Total)
41	1542	1819	<0.1	840	2.17
81	3085	3396	<0.1	840	4.04

of a filter trial design over the frequency range of interest. The 3-D modes are then used in matrix equations to quickly obtain the S-parameters of the filter.

Computations with the new method on two typical low loss bandpass and bandstop filters show good agreement with S-parameter measurements and with computations by the conventional direct frequency FEM. The speedup in computer time increases with increasing numbers of frequencies analyzed; for between 41 and 101 frequencies the speedup over the direct FEM has been found to range from 1.39 to 4.04. Future research in this promising new method will include alternative boundary condition techniques for low loss and high loss microwave problems.

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